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MATHEMATICAL MODEL OF A GLASS-MELTING FURNACE WITH HORSESHOE-SHAPED FLAME

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A method for the construction and implementation of a mathematical zonal model of heat exchange in a glass-melting furnace with a horseshoe-shaped flame and an output of 300 ton/day is considered. The boundary conditions of modeling are described in detail. The authors give the results of the computation of external heat exchange and hydrodynamics of the melting tank, as well as regression equations of temperature fields for refractory brickwork, the flame space, and the glass melt surface.

The practical implementation of the mathematical model of a glass-melting furnace, whose main principles are described in [1], implies the testing of the calculation procedures and the formation of a database of boundary conditions corresponding to the actual parameters of industrial analogs. The furnace selected for testing the methodological algorithm and adapting the mathematical model is the regenerator glass-melting furnace with a horseshoe-shaped flame with an output of 300 ton/day designed by V. Ya. Dzyuzer. The furnace is intended for melting container glass (KT-1, ZT-1, BT-1, etc.) with the option of modifying glass melt tint in the course of its operation. Below are certain specifications of this furnace used as boundary conditions in the development of its mathematical model.

The surface area of the melting tank is 115.77 m^2 (length and width are 13.62 and 8.50 m, respectively). Fuel: natural gas with lower heat-generating capacity Q_{p}^{1} = 35.042 MJ/m^3. The rated thermal load of the furnace for glass ZT-1 for air heating temperature 1200°C and the batch : cullet ratio equal to 70 : 30 is BQ_{p}^{1} = 16.867 MW. Thus, for fuel consumption B = 1732.9 m^3/h the specific heat rate and specific glass melt output are 1159.4 kcal/kg and 2.591 ton/(m^2 · day), respectively.

The concept of the mathematical model of the glass-melting furnace [1] implies a consecutive solution of the exterior and interior problems. Solving the problem of exterior heat exchange in the working space includes several stages.

The first stage is the construction of a zonal model of the furnace and the calculation of the generalized angular radiation coefficient ψ_{ji} (j is the number of the radiating zone and i is the number of the recipient zone). The working space of the furnace and the enclosing brickwork and tank surfaces are split into volumetric (gaseous) and surface zones. The quantity and sizes of zones are selected to register as precisely as possible variations in heat and mass exchange parameters, the tank hydrodynamics, and the stages of the technological process distributed along the melting tank.

It our case the working space of the furnace is represented in the form of a parallelepiped, whose length and width correspond to the real sizes of the industrial analog. The height is selected based on the condition of the equality between the surface areas of the brickwork and the tank: $F_{\text{br}}/F_{\text{t}}$ = const for the furnace and for the model. The working space of the furnace is spilt into five zones along the tank, five zones across its width, and three zones across its height (Fig. 1*a*). The zone coordinates are as follows: along the furnace length $X_0 \to X_5$ = 0, 2.043, 4.086, 7.264, 10.442, and 13.620 m; in width $Y_0 \to Y_5$ = 0, 1.15, 3.35, 5.15, 7.35, and 8.50 m, and in height $Z_0 \to Z_5$ = 0, 0.85, 1.70, and 2.55 m. The limiting surface of the model from below is the glass melt surface.

Thus, the working space of the furnace, including the glass melt surface is represented as 75 volumetric and 110 surface zones. The latter are subdivided into surface zones of types I, II, and III. The surface zones of type I are surfaces with a preset temperatures. These are the surfaces through which air (1350°C) and batch (1450°C) are fed into the furnace and through which the combustion products (1500°C) are removed. The glass melt surface represents surface zones of type II, for which a distribution of heat flows is specified. All enclosing brickwork surfaces are zones of type III, for which a relation between heat flows and temperature is determined.

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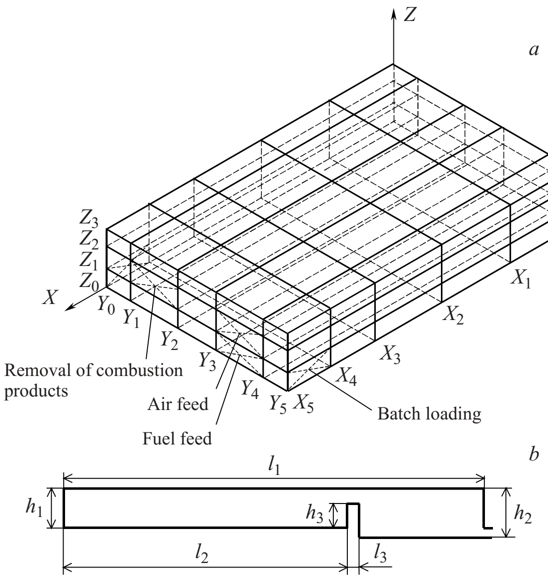

Fig. 1. Approximated zonal model of the working space of the glass-melting furnace with horseshoe flame direction (*a*) and longitudinal section of the melting tank (*b*).

The length l_1 and the width of the melting tank are given above. For tinted glass the depth of the tank before and after the overflow edge is $h_1 = 1.3$ and $h_2 = 1.6$ m, respectively (Fig. 1*b*). The overflow edge of height $h_3 = 0.8$ and width $l_3 = 0.4$ m is placed at a distance of $l_2 = 9.2$ m from the burner inlet. The glass melt moves from the level of the tank bottom through a neck of width 0.8 and height 0.3 m. The model can implement the condition of $h_1 = h_2$.

In calculating ψ_{ji}, an initial radiation characteristic value is set for each zone: attenuation (absorption) coefficient for the volumetric zones and the degree of blackness (reflecting capacity) for the surface zones. The degree of blackness of the brickwork surface zones (roof and walls) was set equal to 0.8. The degree of blackness of the glass melt surface was set in the range of 0.8 – 0.4, decreasing toward the working zone. The radiation absorption coefficient was calculated using the Gurvich – Mitor formula:

$$k_0 = \frac{0.8 + 1.6 p_{H_2O}}{S_{ef}\sqrt{p_{CO_2} + p_{H_2O}}}\left(1 - 0.38\frac{T}{1000}\right);$$

$$S_{ef} = \frac{3.6V}{F},$$

where k_0 is the attenuation coefficient, m^{-1}; p is the partial pressure of the gaseous medium component, Pa; S_{ef} is the effective ray length, m; T is the temperature of the gas (zone), K; V and F are the total volume and surface area of the gaseous zones, m^3 and m^2, respectively.

At the second stage the calculation of the optical-geometric parameters of the zone system elements is completed. The matrix of the coefficients ψ_{ji}, the radiation characteristics of the zones, and data on their mutual position are used to determine the matrices of resolvent generalized angular radiation coefficients f_{ji} by means of solving a system of linear equations of the zonal resolvent method, and then the matrix of radiation exchange coefficients a_{ji} is calculated [1].

At the third stage a database is compiled reflecting the real operating conditions of the furnace, which makes it possible to form a system of nonlinear equations of the resolvent zonal method [1]. Using the restricted jets regularities, the aerodynamic contours and velocities of the flame near the enclosing brickwork and melt surfaces are determined. The aerodynamic contours of direct gas flows (the sequence of passing the zones) and the degree of combustion of fuel are the source data for calculating the mass exchange between the volumetric zones, the combustion process, and the convection exchange coefficients. The current value of the relative air inflow to the combustion zone $\bar{\alpha}$ and the degree of fuel combustion $\bar{\chi}$ are set in the form of the following functions [2]:

$$\bar{\alpha} = \alpha[1 - \exp(-mnx)];$$

$$\bar{\chi} = 1 - \exp(-mx^2),$$

where α is the coefficient of the air flow rate supplied for combustion; $\alpha = 1.1$ (BT-1 and ZT-1), $\alpha = 1$ (KT-1); m, n are the coefficients depending on the flame length; x is the distance from the burner exit section, m.

The particular values of the coefficients m and n are specified based on the following reasoning. It is believed that 85% of the fuel is burnt within the limits of the inflow length l_{in} limited by the value $\bar{\alpha} = 1$. The rest of the fuel is consumed in the afterburning zone. The total length of the flame is $l_f = 1.43 l_{in}$ and the integral chemical underburning of the fuel is equal to $q_3 = 1 - \bar{\chi} = 0.02$ [2]. An adaptation of the model is implemented for $l_{in} = 9.534$ m, which corresponds to the extent of the fuel combustion zone approximately equal to the length of the working space of the furnace: $l_f = 13.634$ m.

For all enclosing brickwork elements the convective heat transfer coefficient $\alpha_c = 8$ W/(m$^2 \cdot$ K). For the glass melt surface zones this coefficient is taken to be constant across the width of the calculation zone and variable along the furnace length: $\alpha_c = 15$, 15, 12, 10, and 8 W/(m$^2 \cdot$ K). The calculations included the following values of the heat capacity of combustion products, air, and fuel: 1.613, 1.425, and 1.571 kJ/(m$^3 \cdot$ K), respectively. The fuel combustion computations determined not only the composition and quantity of the combustion products and the radiation attenuation coefficients, but also the energy parameters of the gas flow. A correct determination of the volumetric density of heat emissions in the gaseous zones depends on the accuracy of the calculation of heat losses via the enclosing brickwork and heat assimilated in the tank.

In the general case the thermal flow via the enclosing surface j is found from the formulas:

$$q_j = K_j\,(T_{\mathrm{br}\,j} - T_{\mathrm{am}});$$

$$K_j = \left(\sum_{i=1}^{n} \frac{S_i}{\lambda_i} + \frac{1}{\alpha_{\mathrm{br}\,j}} \right)^{-1},$$

where q_j is the heat flow via the surface zone j, W/m^2; K_j is the heat transfer coefficient via the surface zone j, W/(m$^2 \cdot$ K); $T_{\mathrm{br}\,j}$ and T_{am} are the temperatures of the inner brickwork surface of the zone j and the ambient medium, K; n is the number of brickwork layers; S_i is the thickness of the ith brickwork layer, m; λ_i is the thermal conductivity of the ith brickwork layer, W/(m$^2 \cdot$ K); $\alpha_{\mathrm{br}\,j}$ is the heat transfer coefficient on the outer brickwork surface, W/(m$^2 \cdot$ K).

For surface zones of type III the heat losses via the enclosing brickwork are taken into account by specifying the temperature of the ambient medium $t_{\mathrm{am}} = 30$°C and the heat transfer coefficient.

Let us consider in more detail the formation of the database for the calculation of K_j. All structural elements of the furnace (the analog) are designed using multilayer refractory insulation. This ensures sufficiently low thermal losses into the ambient medium. The design of a thermally insulated dinas-brick roof and the calculation of heat losses are specified in [3]. It has been demonstrated that for the inner brickwork surface temperature $t_{\mathrm{br}} = 1580$°C and $t_{\mathrm{am}} = 30$°C the effectiveness of the thermal insulation of the roof 400 mm thick (refractory grade DSO) is characterized by the specific heat flows $q_{\mathrm{ex}} = 1437.7$ W/m^2 and the temperature of the exterior brickwork surface $t_{\mathrm{ex}} = 112.2$°C. For a noninsulated roof $q_{\mathrm{ex}} = 5229.7$ W/m^2 and $t_{\mathrm{ex}} = 235.7$°C. It is known that the roof sites adjacent to the temperatures joints and thermocouples are not insulated for safety reasons. This circumstance is taken into account in determining the average value of the heat transfer coefficient for the surface zones of the roof taken as $\bar{K} = 1.29986$ W/(m$^2 \cdot$ K).

In determining K_j the heat losses of the lateral walls of the flame space were taken into account not only as thermal conductance, but also as radiation through the inlet arches, loading hoppers, and peepholes. For this elements of the brickwork $\bar{K} = 4.65036$ W/(m$^2 \cdot$ K). It significantly exceeds the heat transfer coefficient that would have been accepted, if only thermal losses via conductance had been taken into account. For instance, the heat flow through insulated lateral walls made of badelleyite-corundum plates (BK-33) of thickness 200 mm is equal to $q_{\mathrm{ex}} = 1219.5$ W/m^2. The estimated temperature of the external surface of the brickwork insulation is $t_{\mathrm{ex}} = 112.6$°C. For comparison, note that for non-insulated walls $q_{\mathrm{ex}} = 23{,}837.7$ W/m^2 and $t_{\mathrm{ex}} = 527.3$°C [4].

Special attention has been paid to the calculation of the heat transfer coefficient for the walls and bottom of the melting tank. The lateral walls of the tank are made of palisade bars of thickness 250 mm (BK-33, BK-37, and BK-41). The upper part of the bar (200 mm) is not insulated and is subjected to forced air cooling. The remainder part of the bar is arbitrarily split in two parts. The upper part (counting from the melt surface) has intermittent insulation (the bar joints are not insulated) and the lower part has continuous insulation. The cooling intensity of the non-insulated part calculated based on the condition $t_{\mathrm{br}} = 1200 - 1500$°C and $t_{\mathrm{ex}} = 100$°C is characterized by the heat transfer coefficient on the exterior surface of the bar equal to $\alpha_{\mathrm{ex}} = 272.9$ – 365.5 W/(m$^2 \cdot$ K). In this case the specific heat loss is $q_{\mathrm{ex}} = 19{,}114.3 - 25{,}593.1$ W/m^2 [5]. For intermittent insulation $q_{\mathrm{ex}} = 1204.8$ W/m^2 and $t_{\mathrm{ex}} = 111.9$°C. For bare joints $q_{\mathrm{ex}} = 16{,}541.2$ W/m^2 and $t_{\mathrm{ex}} = 449.5$°C. A continuous insulation of the bar ensures $q_{\mathrm{ex}} = 944.7$ W/m^2 and $t_{\mathrm{ex}} = 97.9$°C [4]. The calculations of heat losses for intermittent and continuous insulation are performed for the condition $t_{\mathrm{br}} = 1450$ and 1350°C, respectively.

The design of a heat-insulated bottom of the melting tank and the algorithm of its calculation are considered in detail in [5]. It was shown that the heat losses via the bottom in the melting and clarification zones can be specified as $q_{\mathrm{ex}} = 1130.0$ and 1205.1 W/m^2, respectively. The specified heat flow values correspond to the exterior bottom temperatures of $t_{\mathrm{ex}} = 119.0$ and 123.6°C. The averaged values of heat transfer coefficients for the walls and bottom of the melting tank are taken equal to $\bar{K} = 3.571594$ and 1.02585 W/(m$^2 \cdot$ K), respectively.

For the surface zones of type II representing the glass melt surface, the consumption items of the heat balance were set via the source (discharge) summands of zonal equations [1]. This takes into account the heat consumed in glass formation reactions Q_1', glass melt heating Q_1'', and the compensation of heat losses via the brickwork of the melting tank. Lets us consider some specifics of the calculation of Q_1' and Q_1''. Unlike the heat balance of the furnace, the zonal model has to take into account not only the heat consumed in melting batch, but melting cullet as well. Furthermore, the heat consumed in glass melt heating has to be taken into account. At the same time, the zonal model does not take into account the heat removed from the furnace with the production (working) glass melt flow. In this respect, the known expression for calculating heat consumed in glass formation Q_1' is transformed and has the following form.

The heat consumed (kW) in glass formation can be calculated from the formula:

$$Q_1' = G_{\mathrm{g}}[G_{\mathrm{b}}'(q_{\mathrm{g}} + \sum q_{1.i}) + G_{\mathrm{c}}' q_{\mathrm{c}} - q_{4\mathrm{ph}}],$$

where G_{g} is the furnace efficiency (glass melt output), kg/sec; G_{b} and G_{c} is the consumption of batch and cullet per 1 kg of glass melt, kg/sec; q_{g} is the summary thermal effect of glass formation reactions, kJ/kg; $q_{1.i} = q_{1.1} + q_{1.2} + q_{1.3}$ is the heat consumption on moisture evaporation, heating the degassing products, and glass melting, kJ/kg of batch; q_{c} is

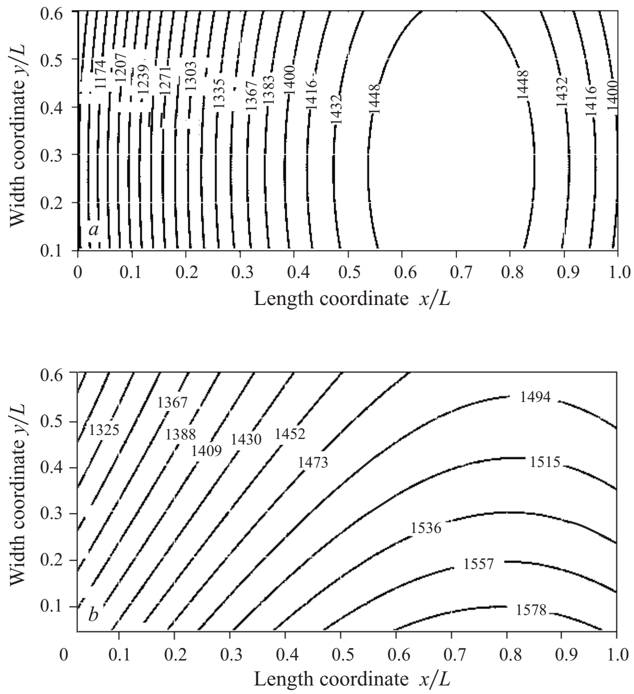

Fig. 2. Temperature field on the surface of the melt (*a*) and roof (*b*). Numbers on the curves indicate glass melt temperature, °C; L is the furnace length equal to 13.62 m.

the heat consumption in melting cullet, kJ/kg; $q_{4\mathrm{ph}}$ is the physical heat of the batch and cullet per the production of 1 kg of glass melt, kJ/kg.

The heat consumption of the glass-formation process determined from the above formula $Q_1' = 3381.51$ kW were distributed along the furnace length in the ratio of 50, 30, 15, 5, and 0%. The heat quantity per calculation zone is distributed across its width in proportion to the surface zone areas. The heat consumed in heating the glass melt up to 1500°C is equal to $Q_1'' = 8233.912$ kW. This component of heat consumption is distributed over all surface zones of the melting tank in proportion to their areas.

The database obtained has made it possible to form a system of nonlinear equations of the resolvent zonal method [1]. Since the unknown quantities in this system are zone temperatures, the equations corresponding to zones of type I were excluded from this system. The results of solving the transformed system of equations are the temperatures of all zones II and III. The calculated temperatures are correlated with the temperatures field used for the calculation of the radiation parameters of the zones. They are taken as final only in the case when the difference between two temperature fields for all zones does not exceed the prescribed calculation error, whose relative value should be not more than 10^{-4}.

The results of calculating surface zone temperatures can be used to form regression equations making it possible to construct isotherm plots for these surfaces. Below are the regression equations of the surfaces of the melt, the roof, and the right and left longitudinal and front walls of the flame space, respectively:

$$t(x, y) = 1132.904 + 931.278x + 67.537y - 675.024x^2 + 4.718xy - 130.01y^2;$$

$$t(x, y) = 1399.65 + 511.682x - 279.713y - 327.564x^2 + 52.049xy + 82.726y^2;$$

$$t(x, z) = 1313.145 + 761.654x + 490.176z - 542.979x^2 - 275.787xz - 1646.567z^2;$$

$$t(x, z) = 1278.097 + 422.825x + 408.297z - 208.091x^2 - 128.850xz - 1567.367z^2;$$

$$t(y, z) = 1575.210 - 14.301y - 350.668z + 24.909y^2 - 361.626yz - 398.718z^2.$$

Figure 2 illustrates the data obtained with the isotherms for the surface of the melt and the roof. They show that the obtained temperature fields quantitatively and distribution-wise correlate well with the real conditions of the furnace with the horseshoe flame direction. The melt surface temperature across its width varies insignificantly. In the end of the batch zone ($x/L \approx 0.3$) it reaches 1350°C. Next, in the foam zone the temperature rise is somewhat slowed. The maximum temperature of 1450 – 1500°C is registered at the quellpunkt ($x/L \approx 0.7$) with a subsequence decrease (by approximately 50°C) toward the front wall of the furnace. The roof temperature typically has a more perceptible heterogeneity across the furnace width. This is due to one-sided flame development and high assimilation of heat in the batch melting zone. Obviously, in transferring the flame the nonuniformity of the roof temperature across the furnace will be leveled. The maximum roof temperature (approximately 1580°C) is registered in the pure glass zone and is located exactly above the overflow edge.

The accumulated data on optical-geometrical and radiation characteristics of zones, radiation and convection exchange coefficients, and zone temperatures make it possible to pass to the next stage of the calculations.

The fourth stage is the estimate of the characteristics of the thermal performance of the furnace, such as specific and integral heat flow densities and the general heat balance of the furnace.

Thus, the results of the third and the fourth stages make it possible to reliably estimate the efficiency of the thermal performance of the furnace. For instance, the total quantity of heat contributed to the working space of the furnace due to the chemical heat of fuel BQ_{p}^{1} and the physical heat of hot air is equal to 25213.00 kW. The processing of the calculation results yielded the following heat distribution among the thermal balance items:

– heat assimilated by the melt surface 11,615.42 kW (46.07%);

– loss via combustion products 11,923.56 kW (47.29%);

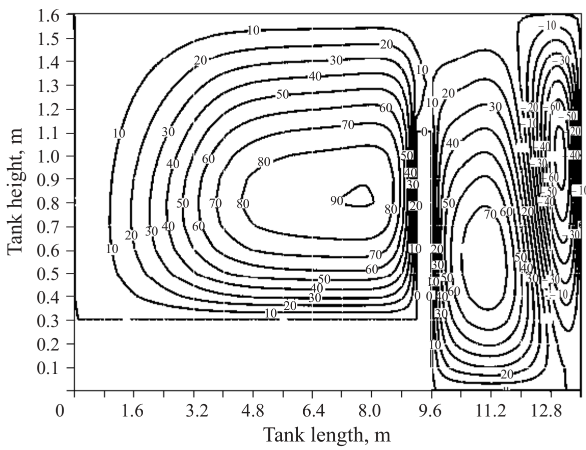

Fig. 3. Distribution of relative value of the flow function in the longitudinal section of the tank.

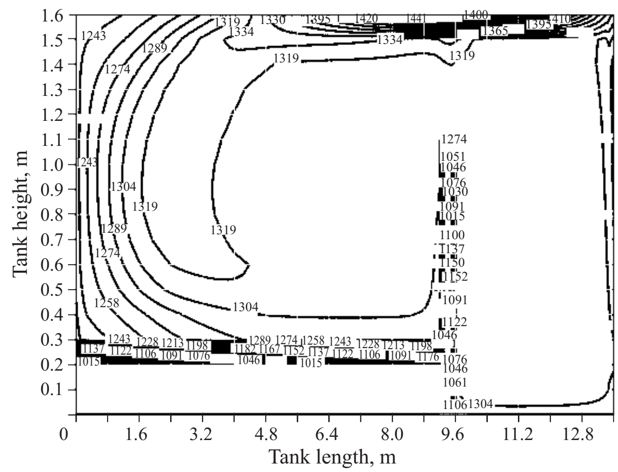

Fig. 4. Temperature distribution in the longitudinal section of the tank. Numbers on the curves indicate glass melt temperature, °C.

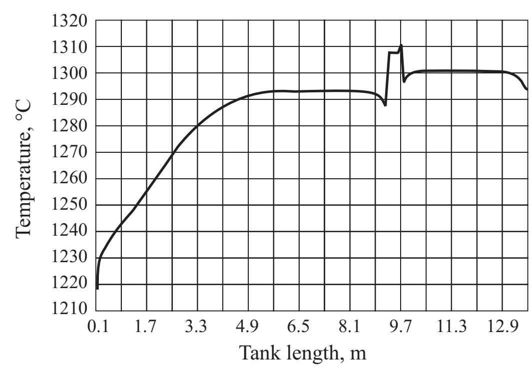

Fig. 5. Distribution of glass melt temperature in the bottom sites along the tank.

– loss via the enclosing brickwork 1337.34 kW (5.30%);
– chemical underburning of fuel 337.36 kW (1.34%).

The furnace efficiency is 68.86%, and the heat utilization coefficient is 46.07%.

The next stage of mathematical modeling is analyzing the correspondence of the external heat exchange parameters to the conditions for producing high-quality glass melt. The initial data for the tank hydrodynamics problem include the temperature field on the melt surface (Fig. 2*a*) and the geometrical parameters of the tank. Computation results make it possible to determine the regularities of the glass melt motion inside the tank (volumetric and mass velocity fields, multiplicity of circulation) and its temperature field. Figure 3 shows the glass melt flow lines in the longitudinal section of the tank and Fig. 4 shows its temperature field in the same section.

According to the definition of the flow function [1], flow lines represent the mass flow rate of the medium in an elementary zone across the tank height. The flow function in Fig. 3. is expressed in fractions of the mass flow rate of molten glass in the furnace neck. Since a steady furnace regime is considered, the flow lines coincide with the trajectories of the medium macroparticles. In other words, Fig. 3 shows the glass melt circulation contours. It can be seen that the glass melt in the tank has a high circulation intensity. This is responsible for an insignificant nonuniformity of the melt temperature field, especially in the tank volume beyond the overflow edge. In the overwhelming majority of the tank volume the temperature difference is not more than 15°C. The average glass melt temperature across the neck section is approximately 1308°C, which corresponds to the conditions of furnaces with a high specific output.

The glass melt temperature fluctuations in the bottom layers of the tank (Fig. 5) appear realistic as well. In the batch melting area the temperature grows in an interval of 1218 – 1290°C. In the foam-formation area due to increasing surface melt temperature and due to glass melt circulation it remains virtually constant at a level of 1293°C. A small temperature decrease in front of the overflow edge (1287°C) is due to the heat loss via the tank bottom. The glass melt temperature on the surface of the overflow edge reaches 1308 – 1310°C, which corresponds to the average temperature of the glass melt in the neck.

Thus, calculation results demonstrate the ample potential of mathematical modeling for investigating particular furnaces and for designing new ones.

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